Numeric Response Questions

Functions

Q.1 If $f: R \to R, f(x) = 2x^3 - 7$, then find pre image of 9 under f.

Q.2 Find the period of the function satisfying the relation $f(x) + f(x+3) = 0 \forall x \in R$.

Q.3 If $f(x)$ is a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, then find the value of n.

Q.4 Find the number of integers lying in the domain of the function $f(x) = \sqrt{\log_{(05)} \left(\frac{5-2x}{x}\right)}$,

Q.5 If the range of the function $f(x) = 6^x + 3x + 6^{-x} + 3^x + 2$ is $[k, \infty)$ then find k.

Q.6 Find period of function $2x + \sin\ \pi x + 3\pi x + \cos\ 2\pi x$ is (where { represent fractional part of x)

Q.7 If period of $\frac{\cos(\sin nx)}{\tan(x/n)} (n \in N)$ is 6π then find value of n.

Q.8 If $f: \mathbb{R} \to \mathbb{R}$ satisfies $f(x+y) = f(x) + f(y)$, for all x, y ∈ R and $f(1) = 7$, and $\sum_{r=1}^{n} f(r) = k(n)(n+1)$ then find k.

Q.9 Let $f(x) = \frac{9^x}{9^x+3}$ and $f(x) + f(1-x) = 1$ then find value of $f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + \cdots \ldots + f\left(\frac{1995}{1996}\right)$,

Q.10 If $f(x) = \begin{cases} 2, & x \in Q \\ -1, & x \in Q \end{cases}$ then find fof(e).

Q.11 If f is a function such that $2f(x) + f(2-x) = x^2$ then find value of $f(4)$.

Q.12 If $f(x)$ is a polynomial function such that $f(x), f(1/x) = f(x) + f(1/x)$ and $f(2) = 9$ then find value of $f(4)$.

Q.13 If the period of $f(x) = \frac{1}{2}\left(\frac{|\sin x|}{\cos x} + \frac{|\cos x|}{\sin x}\right)$ is $k\pi$ then find k.

Q.14 If $f(x) = \cos(\log x)$, then find value of $\frac{f(xy)+f(x/y)}{f(x)f(y)}$.

Q.15 For a real number x, $[.] denotes\ the\ greatest\ integer.$

Find the value of $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \left[\frac{1}{2} + \frac{2}{100}\right] + \cdots . + \left[\frac{1}{2} + \frac{99}{100}\right]$

ANSWER KEY

1. 2.00	**2.** 6.00	**3.** 4.00	**4.** 1.00	**5.** 6.00	**6.** 2.00	**7.** 6.00
8. 3.50	**9.** 997.50	**10.** 2.00	**11.** 9.33	**12.** 65.00	**13.** 2.00	**14.** 2.00
15. 50.00						

Hints & Solutions

1. $2x^3 - 7 = 9 \Rightarrow x = 2$

2. Given $f(x) + f(x + 3) = 0$ ...(1)
Replace x by x + 3,
We have $f(x + 3) + f(x + 6) = 0$
From (1) and (2), $f(x) = f(x + 6)$
Hence, the function has period 6

3. $f(x) = 3^x$ then n = 4

4. $\log_{0.5}\left(\frac{5-2x}{x}\right) \geq 0$

$\Rightarrow \quad 0 < \frac{5-2x}{x} \leq 1 \qquad \Rightarrow \quad x \in \left[\frac{5}{3}, \frac{5}{2}\right)$

5. $6^x + 6^{-x} \geq 2$ and $3^x + 3^{-x} \geq 2$
So $f(x) \geq 6$

6. $f(x) = 2^{\{x\}} + \sin\pi x + 3^{\{x/2\}} + \cos 2\pi x$
Period of $2^{\{x\}}$ is 1
period of $\sin \pi x$ is 2
period of $3^{\{x/2\}}$ is 2
period of $\cos 2\pi x$ is 1
$\therefore$ period of f(x) = L.C.M of these
$\therefore$ period of f(x) is 2

7. $f(x) = \frac{\cos(\sin nx)}{\tan x/n}$
Period of cos (sin nx) is π/n & period of tan x/n is $n\pi$
$\therefore$ period of f(x) is $n\pi$
According to question, $n\pi = 6\pi \Rightarrow n = 6$

8. $f(1) = 7$
$f(x + y) = f(x) + f(y)$
$\Rightarrow x = 1, y = 1$
$\Rightarrow f(2) = f(1) + f(1) = 7 + 7 = 2(7)$
$\Rightarrow x = 2, y = 1$
$\Rightarrow f(3) = f(2) + f(1) = 2(7) + 7 = 3(7)$
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$f(n) = n(7)$

$\Rightarrow \sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + \ldots\ldots + f(n)$
$= 7 + 2(7) + 3(7) + \ldots\ldots + n(7)$
$= 7\,(1 + 2 + 3 + \ldots\ldots + n)$
$= \frac{7n(n+1)}{2}$

9. $f(x) = \frac{9^x}{9^x + 3}$
$\because f(x) + f(1 - x) = 1$
$\therefore f\left(\frac{1}{1996}\right) + f\left(1 - \frac{1}{1996}\right) = 1$
$f\left(\frac{2}{1996}\right) + f\left(1 - \frac{2}{1996}\right) = 1$
$\therefore f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + \ldots + f\left(\frac{1995}{1996}\right) = 997.5$
$\left\{\because f\left(\frac{998}{1996}\right) + f'\left(1 - \frac{999}{1996}\right) = 1 \right.$
$\left. \Rightarrow 2f\left(\frac{998}{1996}\right) = 2 \Rightarrow f\left(\frac{998}{1996}\right) = \frac{1}{2}\right\}$

10. $f \circ f(e) = f[f(e)] = f[-1] = 2$

11. Given $2f(x) + f(2 - x) = x^2$ ..... (i)
substitute x by (2 – x), we get
$2f(2 - x) + f(x) = (2 - x)^2$ ..... (ii)
(1) × 2 – (2), we get
$3f(x) = 2x^2 - (2 - x)^2$

$\Rightarrow f(x) = \frac{1}{3}[2x^2 - (2-x)^2]$

Now, $f(4) = \frac{1}{3}[2(4)^2 - (2-4)^2]$

$= \frac{1}{3}[32 - 4] = \frac{28}{3}$

12. $f(x) = x^n + 1$

$\Rightarrow f(2) = 2^n + 1$

$\Rightarrow 9 = 2^n + 1 \Rightarrow 2^3 = 2^n \Rightarrow n = 3$

clearly, $f(x) = x^3 + 1$

$f(4) = (4)^3 + 1 = 65$

13. by option

$$f(2\pi + x) = \frac{1}{2}\left\{\frac{|\sin(2\pi+x)|}{\cos(2\pi+x)} + \frac{|\cos(2\pi+x)|}{\sin(2\pi+x)}\right\}$$

$$= \frac{1}{2}\left\{\frac{|\sin x|}{\cos x} + \frac{|\cos x|}{\sin x}\right\} = 2\pi$$

14. $f(x) = \cos \log(x)$

$f(y) = \cos \log(y)$

$f(xy) = \cos \log(xy) = \cos(\log x + \log y)$

$f(x/y) = \cos \log(x/y) = \cos(\log x - \log y)$

Now $\frac{f(xy) + f(x/y)}{f(x).f(y)}$

$$= \frac{\cos[\log x + \log y] + \cos[\log x - \log y]}{\cos(\log x).\cos(\log y)}$$

$$= \frac{2.\cos(\log x).\cos(\log y)}{\cos(\log x)\cos(\log y)} = 2$$

$\because \cos(A + B) + \cos(A - B) = 2\cos A \cos B$

15. Do yourself.